HW2 Proof Method

1. Fill in the blanks: given n = 2k for even and n = 2k+1 for odd

* 4 is even because 4=2( 2\_). Here,  4  plays the role of n and  2  plays the role of k .
* 5 is odd because 5=2( 2 \_)+1. Here,  5  plays the role of n and  2  plays the role of k .
* −4 is even because −4=2( -2 \_). Here,  -4 plays the role of n and  -2  plays the role of k .
* −5 is odd because −5=2( -2\_)+1. Here,  -5 \_ plays the role of n and \_-2  plays the role of k .
* 0 is even because 0=2( 0\_). Here, \_2  plays the role of n and \_ 0  plays the role of k .

1. Fill in the blanks Use a **direct proof** to show that If n1 and n2 are even, then n1+n2 is even.

Suppose that n1 and n2 are even. By definition, there exist an integer n1

such that n1=2( k ) and there exists an integer n2 such that n2=2( k ) . Consider the sum of n1 and n2 : n1+n2= 2k+2k =2( 2k ).

Let 2k = 2k Thus n1+n2=2( 2k ) for some integer k . By definition, we know that n1+n2 is even.

1. Use **direct proof** to show the following theorem: If n is even, then n2 is even.

n is even then n = 2k for some integer k.

Consider. n2 = (2k)2

= 4k2

= 2(2k2)

Thus n2 = 2(2k2) for some integer k. By definition, we know that n is even.

1. Use **direct proof** to show the following theorem: If n is an even, then 3n2+5n+18 is even.

n is odd then n = 2k+1 for some integer k.

Consider. 3n2 + 5n + 18 = 3(2k+1)2 + 5(2k+1) + 18

= 3(4k2 + 4k +1) + 10k + 5 + 18

= 12k2 + 12k +3 + 10k +5 + 18

= 12k2 + 22k + 26

= 2(6k2 + 11k + 13)

Thus 3n2 + 5n + 18 = 2(6k2 + 11k + 13) is even. for some integer k. By definition, we know that n is

1. Fill in the blanks. Use a **contrapositive proof** For any integer n, if 5n+3 is even, then n is odd.

If n is not odd, 5n+3 is not even

Suppose that n is not odd. We know that n is even . By definition, there exists an integer such that n=2( k ) . Consider:

5n+3= 5(5k)+3 = 10k+3 =2( 5k+1 )+1.

Let 2k = 2(5k+1)+1 . Thus 5n+3=2( 5k+1 )+1

for some integer k . By definition, we know that 5n+3 is odd. Hence 5n+3 is not even.

1. Use a **contrapositive proof** If  3n2+5n+18 is even, then n is an even.

If n is even, then 3n2+5n+18 is even.

Suppose that n is not even. We know that n is even . By definition, there exists an integer

such that n = 2k.

Consider:

3n2+5n+18 = 3(2k)2+5(2k)+18

= 12k2 + 10k +18

= 2(6k2 + 5k + 9)

Thus 3n2+5n+18 = 2(6k2 + 5k + 9) for some integer k.

By definition, we know that 3n2+5n+18 is even.

Hence 3n2+5n+18 is even.

1. Use a **contrapositive proof** If n2is even, then n is even.

contrapositive, ¬Q → ¬p

If n is not even, then n2 is not even.

Suppose that n is not even. We know that n is odd .

By definition, there exists an integer such that n=2k+1 .

Consider: n2 = (2k+1)2

= 4k2 + 4k +1

= 2(2k2 + 2k) +1

Thus n2 = 2(2k2 + 2k) +1 for some integer k. By definition, we know that n2 is odd. Hence n2 is not even.

1. Fill in the blanks. **Use proof by contradiction**

Both p and ¬q are true

Proof. Suppose, for the sake of contradiction, that n2 + 5 is odd and n is also odd .

By definition, then, there exists integers k and l so that n2 + 5 = 2k+1 and n = 2k+1 .

Hence, we have

2k + 1 = n2 + 5

= ( 2k+1 )2 + 5

= 4k2+4k+1 + 5

= 2( 2k2+2k+3 ).

Therefore, 2k + 1 is (2k2 + 2k + 3) . This is clearly impossible, and hence we cannot have that n2 + 5 is odd and n is also odd.

Therefore, if that n2 + 5 is odd, we must have n is even.

1. Use **proof by contradiction** to show that if n is an integer, and n3+5 is odd, then n is even

Both Q and ¬Q are true

Suppose, for the sake of contradiction, that n3 + 5 is odd and n is also odd .

By definition, then, there exists integers k so that n2 + 5 = 2k+1 and n = 2k+1 . Hence, we have

2k + 1 = n3 + 5

= (2k + 1) + 5

= (8k3 + 12k2 + 6k + 1) + 5

= 8k3 + 12k2 + 6k + 6

= 2(4k3 + 6k2 + 3k + 3)

Therefore, 2k+1 is 2(4k3 + 6k2 + 3k + 3). This is clearly impossible, and

hence we cannot have that n3 + 5 is odd and n is also odd.

Therefore, if that 3n2+5n+18 is odd, we must have n is even.

1. Use **proof by contradiction** to show that if 3n2+5n+18 is even, then n is an even

Both Q and ¬Q are true

Proof. Suppose, for the sake of contradiction, that 3n2+5n+18 is odd and n is even.

By definition, then, there exists integers k so that 3n2+5n+18 = 2k+1 and n = 2k.

Hence, we have

2k+1 = 3n2+5n+18

= 3(2k)2 + 5(2k) + 18

= 3(4k2) + 5(2k) + 18

= 12k2 + 10k + 18

= 2(6k2 + 5k +9)

Therefore, 2k+1 is 2(6k2 + 5k + 9). This is clearly impossible, and hence we cannot have that 3n2

+5n+18 is odd and n is even.

Therefore, if that 3n2+5n+18 is odd, we must have n is not even